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## CONTROL OF AN INDUSTRIAL COPPER SOLVENT EXTRACTION PROCESS

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### Abstract

A two level control strategy that stabilizes and optimizes the production of an industrial copper solvent extraction process is presented. The stabilizing layer consists of a multi-input multi-output controller or two single-input single-output controllers with additional four feedforward compensators that regulate the flow rates in the copper solvent extraction process. The optimization layer consists of an optimizer that maximizes the production of the copper solvent extraction process and gives setpoints to the controllers at the stabilizing level. The mechanistic plant models, verified with industrial data, are linearized by identifying first and higher order transfer function models from simulated PRBS data. On the basis of the linear models, the interactions of the controlled variables, and the pairing of the controlled and manipulated variables are studied and the optimizer and the controllers designed. The control strategy employing two PI-control loops or a model predictive controller and additionally four feedforward control loops is successfully tested against simulated disturbances and setpoint changes. The control strategy is also compared to the data collected from the industrial plant under manual control. With this two level control strategy the production of the copper solvent extraction process is increased by 3-5% and the process variation is decreased by 70-90% compared to the manual operation of the case industrial plant. The results gained in simulation environment are successful and encouraging for further testing in an industrial plant. *Copyright* © 2006

### Keywords

Industrial control, copper solvent extraction, SX/EW, hydrometallurgy, extractive metallurgy, control, PI controller, model predictive control, control application.

## 1 Introduction

The copper leaching, solvent extraction and electrowinning (LX/SX/EW) process is one of the most important methods in the production of copper from low-grade oxidized ore. Research on the copper LX/SX/EW process to date has focused on the process chemicals and equipment, and control of the industrial plants relies on simple PID control loops and manual tuning. Advanced control of a solvent extraction plant would make it possible to keep the process variables closer to their optimal values, thus increasing the amount of copper produced, and reducing the amount of chemicals and energy consumed [1-3].

Very few papers are published on the modelling and control of industrial copper solvent extraction processes. The steady state of a copper solvent extraction process can be studied with mass balance equations and equilibrium diagrams, for example equilateral-triangular diagrams and McCabe-Thiele diagrams. McCabe-Thiele diagram is widely used in the industry for the plant design and production optimization. [4-6]

A steady state model of an industrial copper solvent extraction and electrowinning plant was developed by Aminian et al [7]. The model for the solvent extraction considers mass transfer of copper and iron in the mixers. The mixer model is based on mass conservation and it includes both transfer to the interfacial surface of the phases and the reaction rate

over the surface. The equilibrium curves are experimentally defined. The steady state model of the electrowinning is based on the basic formulation of electrolysis reaction. The models were combined to agree with the flow sheet of the pilot plant consisting of two series extraction units, one stripping unit and an electrowinning process. The measurements and steady state model predictions were compared with good agreement.

The structure of a continuous dynamical model for mixer-settler cascade was summarized by Wilkinson and Ingham [8]. The model for one extraction step consisted of an ideal mixing model describing a mixer and a plug flow model describing a settler. The underlying assumptions were: (1) perfect mixing in the mixer, (2) equilibrium is immediately achieved in the mixer, (3) the aqueous and the organic solutions are immiscible, (4) flow rates for both phases are constant, (5) plug flow separately for both phases in the settler, (6) no mass transfer in the settler. The equilibrium was calculated from linear equilibrium curve or by using Murphree efficiency. The mass transfer coefficient was derived from two film theory using the phase specific film coefficients.

Ingham et al. [9] suggested modifying the mixing model by adding an entrainment flow to the mixer settler model. If the phase separation is imperfect, the aqueous flow out of the settler contains organic droplets and the organic flow out of the settler aqueous droplets.

Hoh et al. [10] have modeled copper solvent extraction by a continuous-flow stirred tank reactor model. The model considers the copper, reagent and hydrogen ion concentrations together with the flow rates. The mass transfer coefficient is modeled as function of the aqueous phase hold up, and the equilibrium constant is determined experimentally. The model fit to the experimental data, for laboratory system with one extraction step, one washing step and five stripping steps, was very good.

To date, no advanced control strategies or systems have been reported in literature or known to be utilized in the plants. The control of the copper solvent extraction and electrowinning processes relies on operators actively manipulating the setpoints for the basic level controllers. This means that the control loops are decentralized with manual setpoints for flow rates, levels, motor speeds, stirring speeds, pumps and valves. Since the process includes long time delays and complex interactions between the variables, the optimal performance and productivity of the plant is seldom achieved. [11, 1, 12]

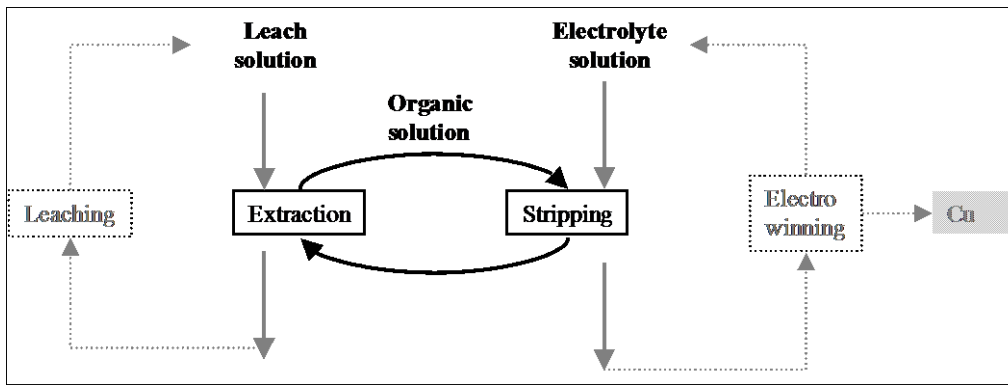
In this paper, an industrial solvent extraction process is described and the mechanistic model (Komulainen et al. [13]) is briefly reviewed. Then the control strategy is described, and the optimization and controller design are presented. Finally the control results are presented and discussed.

## **2 Process description**

The aim of the copper solvent extraction process is to concentrate aqueous copper solution from a few g/l to about 40 - 50 g/l and to purify the solution from ferrous, manganese, chloride and other impurities, which are harmful for the downstream electrowinning process. This continuous process consists of extraction and stripping processes, both of which may contain several unit operations. In the extraction units, copper is transferred from the mildly acid aqueous leach solution to the organic solution. In the stripping units, copper is transferred from the organic solution to the strongly acid aqueous electrolyte solution. In industry, the process equipment is solely mixer-settlers. In the mixers, the minor phase is dispersed to the major phase, and the copper ion transfer takes place on the droplet interphase between the phases. The dispersed phases are separated by gravity in the long and shallow settlers. [14, 2, 15]

Generally, the process has two input flows, the pregnant leach solution (PLS) and the lean electrolyte (LE), and one recycling flow, the organic solution. In the extraction units, copper is extracted from the PLS to the barren organic (BO) solution. In the stripping units, copper is stripped from the loaded organic solution (LO) to the lean electrolyte (LE) solution. The result of stripping, i.e., the rich electrolyte solution (RE), is blended and fed to the electrowinning process, where 99.99% pure copper cathodes are produced. A flow diagram of the process is shown in Fig.

Typical process instrumentation includes flow rate, temperature, and level measurements. In addition, on-line measurements of conductivity in the mixers, pH, and copper and impurities may be included. The copper and impurity assays, phase ratio, phase separation time and other diagnostic measurements can be measured offline in the laboratory.



**Fig. 1.** Flow diagram of the copper solvent extraction process. In the extraction units, copper is extracted from the pregnant leach solution to the organic solution. In the stripping units, copper is stripped from the loaded organic solution to the electrolyte solution. The resulting rich electrolyte solution is led to the electrowinning process. The barren organic solution is recycled back to the extraction units.

The main control targets of the copper solvent extraction process are to maximize the extracted copper and to provide rich electrolyte with stable concentration to the electrowinning process. To achieve these goals, the throughput has to be maximized and the disturbances rejected by balancing the flow rates and adjusting the chemical levels (acidity of aqueous solutions and reagent concentration in the organic solution).

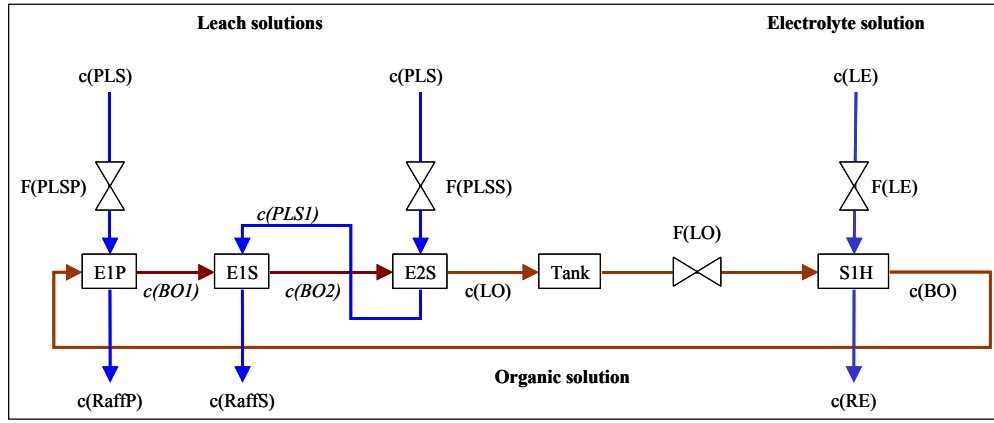
In the studied plant, both flow rates and chemical concentrations are currently adjusted manually. Since automatic instrumentation is available only for the flow rates, the control of chemical concentrations is excluded from this study. The system requires on-line measurements of the flow rates and the copper concentrations.

### 3 Model description

#### 3.1 Mechanistic process model

The studied copper solvent extraction process consists of four units, three for extraction and one for stripping, as presented in Fig. 2. The inputs of the process are pregnant leach solution series and parallel flow rates,  $F(PLSS)$  and  $F(PLSP)$ , pregnant leach solution copper concentration,  $c(PLS)$ , lean electrolyte flow rate,  $F(LE)$ , lean electrolyte copper concentration,  $c(LE)$ , and the flow rate of the organic solution,  $F(LO)$ . The organic solution is recycled in the process, but the flow rate can be manipulated through the organic storage tank. The outputs of the process are rich electrolyte copper concentration,  $c(RE)$ , and raffinate copper concentrations,  $c(RaffS)$  and  $c(RaffP)$ . The process states include the concentrations of the organic solution after each unit operation, and the organic to aqueous volume ratios. The laboratory measurements of the reagent volume percent in the organic solution,  $vol$ , the pH level in the leach solution,  $PH$ , and the acidity of the electrolyte solution,  $Acid$ , are used to update the plant specific McCabe-Thiele equilibrium isotherm parameters. These offline measured variables vary very slowly, and, therefore interpolation between the samples is considered giving accurate enough estimates.

In the control context, the controlled variables (CV) of the process are the rich electrolyte copper concentration,  $c(RE)$ , and the loaded organic copper concentration,  $c(LO)$ . The flow rates are manipulated variables (MV) and the PLS and lean electrolyte copper concentrations are (measured) disturbance variables (DV). The process variables and indication of the measurement type (online/offline) are presented in Table 1.



**Fig. 2.** The case copper solvent extraction process. The output variables are the copper concentrations of the raffinates  $c(\text{RaffP})$  and  $c(\text{RaffS})$ , rich electrolyte,  $c(\text{RE})$ , and the copper concentrations of the recycled organic stream, barren organic,  $c(\text{BO})$  and loaded organic  $c(\text{LO})$ . The input variables are the flow rates of PLS,  $F(\text{PLSS})$  and  $F(\text{PLSP})$ , organic,  $F(\text{LO})$ , and lean electrolyte,  $F(\text{LE})$ , and the copper concentration of PLS,  $c(\text{PLS})$  and lean electrolyte,  $c(\text{LE})$ .

**Table 1**

Controlled, manipulated, disturbance and state variables of the industrial copper solvent extraction process. The measurement type, online/offline, is indicated on fourth and fifth column, with an indication of the online and offline measurements.

Classification	Variable name	Abbreviation	Online	Offline
Controlled variables	Rich electrolyte copper concentration	$c(\text{RE})$	X	X
	Loaded organic copper concentration	$c(\text{LO})$	X	X
Manipulated variables	PLS series flow rate	$F(\text{PLSS})$	X	
	PLS parallel flow rate	$F(\text{PLSP})$	X	
	Organic flow rate	$F(\text{LO})$	X	
	Electrolyte flow rate	$F(\text{LE})$	X	
Disturbance variables	PLS copper concentration	$c(\text{PLS})$	X	X
	Lean electrolyte copper concentration	$c(\text{LE})$	X	X
	Reagent volume percent in organic solution	Vol		X
	pH level of the PLS solution	PH		X
	Acidity of electrolyte solution	Acid		X
State variables	Partial PLS copper concentration	$c(\text{PLS1})$		X
	Partial organic copper concentration after E1P step	$c(\text{BO1})$		X
	Partial organic copper concentration after E1S step	$c(\text{BO2})$		X
	Raffinate series copper concentration	$c(\text{RaffS})$	X	X
	Raffinate parallel copper concentration	$c(\text{RaffP})$	X	X
	Barren organic copper concentration	$c(\text{BO})$	X	X

In this work, the solvent extraction process is modeled considering only the mass transfer of copper between organic and aqueous phases. Each unit process is modeled as combination of an ideal mixer and settler with on-line estimated parameters. The model is described in more detail in Komulainen et al. [13].

The copper transfer is calculated from the ideal mixing equations, where the equilibrium value  $c^*$  is determined on the basis of the incoming flow rates, concentrations, efficiency coefficients and plant specific McCabe-Thiele diagram. The variables are marked: flow rates  $F$ , the mixing volumes  $V_{\text{mix}}$ , organic concentrations  $c^{\text{org}}$ , aqueous concentrations  $c^{\text{aq}}$ , mass transfer coefficients  $K$ , efficiency parameters  $\alpha$ . The mixer are treated as ideal mixers with adapted parameters  $K$ ,  $\alpha$ , and equilibrium isotherm coefficients,  $A$  and  $B$  for extraction, and  $C$  and  $D$  for stripping. The settler, always following the mixer, is described by a pure time delay and marked with  $t_i$ .

In extraction, copper is transferred from the aqueous leach solution to the barren organic solution. An extraction unit is modelled with Eq. (1), where the equilibrium is calculated at each time step from nonlinear Eq. (2). Each of the three extraction steps can be formulated as follows:

$$\frac{dc_i^{\text{org}}(t)}{dt} = \frac{F_i^{\text{org}}(t)}{V_{\text{mix},i}} \cdot [c_{i-1}^{\text{org}}(t - t_{i-1}) - c_i^{\text{org}}(t)] + K_i[c_i^{\text{org}}(t) - c_i^{\text{org}*}(t)] \quad (1)$$

$$c_i^{org*}(t) = g(c_{i-1}^{org}(t-t_{i-1}), c_{i-1}^{aq}(t-t_{i-1}), F_i^{org}(t), F_i^{aq}(t), \alpha_i, A, B) \quad (2)$$

Loaded organic copper concentration, the second controlled variable, is the time delayed value of the output of the third extraction Eq. (1). Loaded organic copper concentration is defined as follows:

$$c(LO)(t) = c_3^{org}(t-t_3) \quad (3)$$

In the stripping, copper is transferred from the loaded organic solution to the lean electrolyte solution resulting in a rich electrolyte solution. The stripping unit process is modeled with Eq. (4), where the equilibrium is calculated from linear Eq. (5). The stripping unit operation is modeled as follows:

$$\frac{dc_1^{el}(t)}{dt} = \frac{F_1^{el}(t)}{V_{mix4}(t)} \cdot [c_0^{el}(t-t_4) - c_1^{el}(t)] + K_4 [c_1^{el}(t) - c_1^{el*}(t)] \quad (4)$$

$$c_1^{el*}(t) = h(c_3^{org}(t-t_3), c_0^{el}(t), F_4^{org}(t), F_1^{el}(t), \alpha_4, C, D) \quad (5)$$

Rich electrolyte copper concentration, that is the first controlled variable, is the time delayed value of the output of the stripping Eq. (4). Rich electrolyte copper concentration is defined as follows:

$$c(RE)(t) = c_1^{el}(t-t_4) \quad (6)$$

The nonlinear model structure with adaptation in the equilibrium isotherm and efficiency parameters performed very well in comparison to industrial process data, and therefore was chosen for the control studies. For the model details, please refer to Komulainen et al. [13]. The model was simulated in Matlab, using the ode15s stiff solver for integration.

### 3.2 Linear model identification

In order to develop linear controllers for the process, the mildly nonlinear mechanistic model was first linearized to first and higher order plus time delay transfer function model and to linear state space model form. The output variables are rich electrolyte, loaded organic copper concentrations,  $c(RE)$  and  $c(LO)$  (see Table 1). The input variables are the flow rates of the PLS series, PLS parallel, organic and electrolyte,  $F(PLSS)$ ,  $F(PLSP)$ ,  $F(LO)$  and  $F(LE)$ , the copper concentrations of PLS and lean electrolyte,  $c(PLS)$  and  $c(LE)$ , and the reagent volume percent in the organic solution,  $vol$ . The input variable vector  $[F(PLSS) F(PLSP) F(LO) F(LE) c(PLS) c(LE) vol]$  is referred as  $[IN]$  and the output vector  $[c(LO) c(RE)]$  is referred as  $[OUT]$  in this paper.

The state space model for the outputs, the loaded organic and rich electrolyte copper concentrations,  $c(LO)$  and  $c(RE)$ , is defined as follows:

$$\begin{aligned} [\dot{x}] &= M_1[x] + M_2[IN]^T \\ [OUT]^T &= M_3[x] + M_4[IN]^T \end{aligned} \quad (7)$$

where the number (n) of the states  $x$  determines the order of the coefficient matrices  $M_1 \in \mathbb{R}^{n \times n}$ ,  $M_2 \in \mathbb{R}^{n \times 7}$ ,  $M_3 \in \mathbb{R}^{2 \times n}$ . The matrix  $M_4$  in this case is  $[0]$  since there are no direct effects from the inputs to the outputs.

In the transfer function model most of the submodels are first order plus time delay form. The more complex dynamics are modeled as second order with zero plus time delay. The transfer function models for the outputs, the loaded organic and the rich electrolyte copper concentrations are of the following form:

$$[OUT]^T = \begin{bmatrix} \frac{a_{11}e^{-c_{11}s}}{b_{11}s+1} & \frac{a_{12}e^{-c_{12}s}}{b_{12}s+1} & \frac{a_{13}e^{-c_{13}s}}{b_{13}s+1} & \frac{a_{14}e^{-c_{14}s}}{b_{14}s+1} & \frac{a_{15}e^{-c_{15}s}}{b_{15}s+1} & \frac{a_{16}e^{-c_{16}s}}{b_{16}s+1} & \frac{a_{17}e^{-c_{17}s}}{b_{17}s+1} \\ \frac{a_{21}e^{-c_{21}s}}{b_{21}s+1} & \frac{a_{22}e^{-c_{22}s}}{b_{22}s+1} & \frac{a_{23}(d_{23}s+1)e^{-c_{23}s}}{(b_{231}s+1)(b_{232}s+1)} & \frac{a_{24}e^{-c_{24}s}}{b_{24}s+1} & \frac{a_{25}e^{-c_{25}s}}{b_{25}s+1} & \frac{a_{26}e^{-c_{26}s}}{b_{26}s+1} & \frac{a_{27}(-d_{27}s+1)e^{-c_{27}s}}{(b_{271}s+1)(b_{272}s+1)} \end{bmatrix} [IN]^T \quad (8)$$

The state space model matrices and transfer function model parameters were identified from the simulated pseudo-random binary sequence data using Matlab system identification toolbox (for the N4SID identification algorithm, see Ljung [16]). Next, the models were validated by applying the inputs of the validation data sets to the linear models and comparing the linear model outputs to the outputs of the validation data sets.

The model performances are compared to each other with the fitness index. The fitness index is the percentage of the output variations that is reproduced by the model, the higher the percentage is, the better fit. (Matlab system identification toolbox: Ljung [16]).

$$fit = \left[ 1 - \frac{norm(Y_{meas} - Y_{model})}{norm(Y_{meas} - \bar{Y}_{meas})} \right] \cdot 100\% \quad (9)$$

where  $Y_{meas}$  is the measured value,  $\bar{Y}_{meas}$  is the average value of the measured outputs,  $Y_{model}$  is the model value. The fit index values are between -100% and 100%, where 100% represents perfect fit.

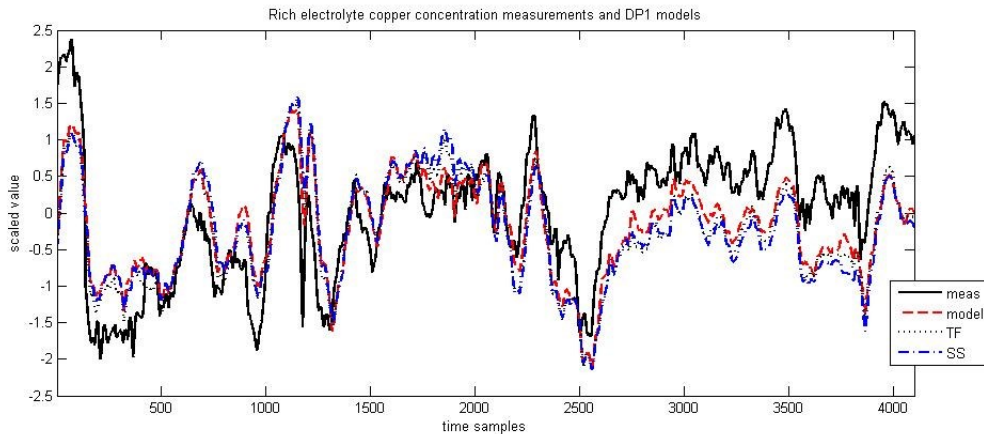
The linear models were validated against three different data sets, one input step change response data set, one simulated data set, and one industrial data set. The fit indices are presented in Table 2, and visual comparison for the rich electrolyte copper concentration in **Error! Reference source not found.**. The model linearization was successful with both model structures, the best fit indices were for the transfer function models and state space models of 8<sup>th</sup>- and 10<sup>th</sup>- order. The transfer function and the eight order state space models are used in the further control studies.

**Table 2**

Model fits to the rich electrolyte (c(RE)) and loaded organic (c(LO)) copper concentration responses to the 5% input steps (Valid 1) at the first operating point, to mechanistic model outputs (Valid2) with the inputs of the first industrial data set, and to the first industrial data set (Valid3).

Model structure	Valid1 c(RE)	Valid2 c(RE)	Valid3 c(RE)	Valid1 c(LO)	Valid2 c(LO)	Valid3 c(LO)
Transfer function (Eq. 10)	89.64	79.34	27.19	93.82	39.23	4.837
State space 2 order	67.51	69.7	20.36	60.01	31.27	16.05
State space 3 order	62.89	57.94	23.33	55.71	30.59	16.22
State space 4 order	59.07	53.23	28.59	51.47	30.6	16.47
State space 5 order	70.33	70.2	20.35	52.79	30.04	16.77
State space 6 order	68.17	66.8	25.87	57.84	36.2	15.63
State space 7 order	87.37	75.35	14.29	67.72	39.72	13.09
State space 8 order	91.7	72.96	13.21	75.42	41.6	10.24
State space 10 order	90.9	76.92	16.7	75.43	40.69	10.45
State space 12 order	90.66	78.03	17.74	73.91	41.69	10.39
State space 14 order	92.75	75.77	15.97	74.6	41.72	9.783

For example, compared to the rich electrolyte copper concentration measurements of the first industrial data set (Valid3), the linear transfer function and eight order state space models follow well the dynamics of the process, as can be seen from **Error! Reference source not found.**.



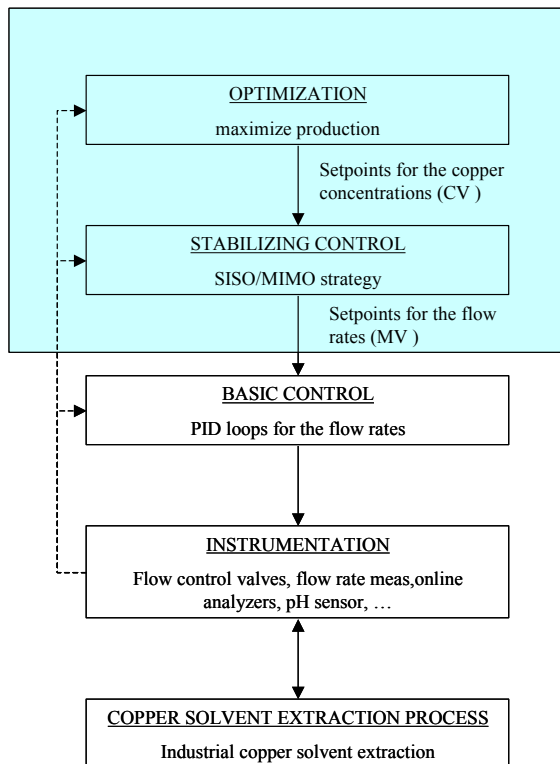
**Fig 3.** Rich electrolyte copper concentration; measurement of the first industrial data set (solid), mechanistic model (dashed), transfer function model (dotted), and eight order state space model (dash dotted).

## 4 The proposed control strategy

### 4.1 Design of the control strategy

A two level control strategy that stabilizes and optimizes an industrial copper solvent extraction process is developed. The three level control strategies are illustrated in Fig, where the two highest levels, optimization level and stabilization level, are included in this study. The stabilization layer consists of a multi-input multi-output controller or two single-input single-output controllers with four feedforward compensators that regulate the flow rates in the copper solvent extraction process. The optimization layer consists of an optimizer that maximizes the production of the copper solvent extraction process.

In the optimization layer, the production of the copper solvent extraction process is maximized on the basis of the linearized, dynamic model of the copper solvent extraction process. The optimization layer provides setpoints for the controlled variables (copper concentrations) at the stabilizing control level. At the stabilizing control level the single-input single-output controllers or alternatively a multi-input multi-output controller are keeping the controlled variables at the given setpoints by manipulating the flow rate setpoints. The flow rate setpoints are further given to the basic level controllers. The stabilizing control level consists of two PI controllers or a model predictive controller and additionally four feedforward compensators, which are designed on the basis of the linearized, dynamic model of the copper solvent extraction process. At the basic control level, the flow rate controllers (PID controllers) are manipulating the opening of the control valve by comparing the difference between the flow rate setpoint and the flow rate measurement. The measurement information (flow rates, copper concentrations) is led to the higher control levels.



**Fig. 4.** Proposed control hierarchy for the copper solvent extraction process. The local optimization layer provides the setpoints of the controlled variables to the supervisory control level. The supervisory control level is based on single-input single-output or multi-input multi-output control strategy. The supervisory control level provides the setpoints of the manipulated variables to the regulatory control level. The regulatory control level gives signals to the final control elements in the instrumentation level. The measurement information is led from the instrumentation level to all the upper levels.

The controlled variables (CV) are the output of the extraction, the loaded organic copper concentration ( $c(LO)$ ), and the output of stripping, the rich electrolyte copper concentration ( $c(RE)$ ). The available manipulated variables (MV) are the flow rates of PLS, organic and electrolyte,  $F(PLSS)$ ,  $F(PLSP)$ ,  $F(LO)$  and  $F(LE)$ . The measured disturbance variables (DV) are the PLS and lean electrolyte concentrations, and the total PLS flow rate, the unmeasured disturbance is the change in reagent volume percent in the organic solution and pH changes in PLS and acidity changes electrolyte solution. The restrictions are the organic to aqueous ratio, which is related to phase continuity, pumping capacity, and the organic level in the tanks.



#### 4.2 Design of the optimization algorithm

The optimization provides setpoints of the controlled variables for the stabilizing layer controllers, and is the local optimization level of the proposed control hierarchy. The setpoints of the controlled variables and the optimal values of the manipulated variables are determined by solving the linear optimization problem with the linear constraints. The optimal values of the controlled and manipulated variables are calculated on the basis of the values of the disturbance variables and the optimization parameters.

The maximization of the production, i.e. the copper mass flow out of the copper solvent extraction process, can be formulated mathematically as the copper concentration difference between the rich and lean electrolytes times the electrolyte flow rate, as follows:

$$P = [c(RE) - c(LE)]F(LE) \quad (10)$$

The restrictions of the optimization problem are the aqueous to organic ratios in the mixers. In the extraction part the mixers are assumed to run aqueous continuous, i.e. the major phase is aqueous, and therefore the organic to aqueous ratio has to be below  $\beta_1$  ( $<1$ ). In the stripping part, the mixers are assumed to run organic continuous with organic to aqueous ratio above  $\beta_2$  ( $>1$ ). These restrictions can be formulated for the extraction as follows:

$$F(LO) \leq \beta_1 \cdot F(PLSS) \quad F(LO) \leq \beta_1 \cdot F(PLSP) \quad (11)$$

and for the stripping as follows:

$$F(LO) \geq \beta_2 \cdot F(LE) \quad (12)$$

The optimization problem can be presented with the controlled, manipulated and disturbance variables by assuming that the transfer function models of the plant present the steady state of the process adequately well. Now, the rich electrolyte copper concentration can be presented as:

$$c(RE) = [w_1 \ w_2 \ w_3 \ -w_4 \ w_5 \ w_6 \ w_7][IN]^T \quad (13)$$

and the loaded organic copper concentration can be presented as:

$$c(LO) = [r_1 \ r_2 \ -r_3 \ -r_4 \ r_5 \ r_6 \ r_7][IN]^T \quad (14)$$

The constants,  $w_i$  and  $r_i$ , are positive, and represent the absolute values of the steady state gains of the transfer function models for the loaded organic and rich electrolyte copper concentrations. On the basis of these equations, the profit function (10) can be expressed with the manipulated and disturbance variables as follows:

$$P = [w_1 \ w_2 \ w_3 \ 0 \ w_5 \ -(1 - w_6) \ w_7][IN]^T \cdot F(LE) - w_4 F(LE)^2 \quad (15)$$

The maximization of this equation requires maximum value for the organic flow rate,  $F(LO)$ . The maximum values of the manipulated flow rates can be derived from the restrictions of the optimization problem. The organic flow rate has a maximum restriction in relation to the minimum of the PLS flow rates. Since maximum value for the organic flow rate is desired, Eq. (11) yields:

$$F(LO)_{opt} = \beta_1 \cdot \min\{F(PLSS), F(PLSP)\} = \beta_1 \cdot F(PLS)_{min} \quad (16)$$

where the minimum of the two PLS flow rates, PLSS and PLSP, is marked as  $F(PLS)_{min}$ .

The electrolyte flow rate,  $F(LE)$ , has an optimum point, which can be calculated by setting the derivative of the profit function (with the optimal organic flow rate) to zero. The derivative is taken in relation to the electrolyte flow rate. The optimal electrolyte flow rate is now defined as follows:

$$F(LE)_{opt} = \frac{1}{2w_4} [w_1 \ w_2 \ \beta_1 w_3 \ 0 \ w_5 \ -(1 - w_6) \ w_7] \cdot [F(PLSS) \ F(PLSP) \ F(PLS)_{min} \ F(LE) \ c(PLS) \ c(LE) \ vol]^T \quad (17)$$

This is the optimal value for the electrolyte flow rate if the maximum electrolyte flow rate limitation is not exceeded, as required in Eq. (12). The maximum for the electrolyte flow rate is smaller than the optimum organic flow rate divided by  $\beta_2$ :

$$F(LE)_{optF} = \min \left\{ F(LE)_{opt}, \frac{1}{\beta_2} F(LO)_{opt} \right\} \quad (18)$$

Now the setpoints for the rich electrolyte and loaded organic copper concentration can be formulated on the basis of the optimum manipulated variables, the disturbance variables and optimization parameters as follows:

$$cRE_{sp} = [w_1 \ w_2 \ -\beta_1 w_3 \ -w_4 \ w_5 \ w_6 \ w_7] \cdot [F(PLSS) \ F(PLSP) \ F(PLS)_{min} \ F(LE)_{optF} \ c(PLS) \ c(LE) \ vol]^T \quad (19)$$

$$cLO_{sp} = [r_1 \ r_2 \ -\beta_1 r_3 \ -r_4 \ r_5 \ r_6 \ r_7] \cdot [F(PLSS) \ F(PLSP) \ F(PLS)_{min} \ F(LE)_{optF} \ c(PLS) \ c(LE) \ vol]^T \quad (20)$$

The maximal production can be calculated by substituting Eqs. (16) and (18) to Eq. (10) as follows:

$$P = [w_1 \ w_2 \ \beta_1 w_3 \ -w_4 \ w_5 \ -(1-w_6) \ w_7] \cdot [F(PLSS) \ F(PLSP) \ F(PLS)_{min} \ F(LE)_{optF} \ c(PLS) \ c(LE) \ vol]^T \cdot F(LE)_{optF} \quad (21)$$

#### 4.3 Design of siso and mimo controllers

The pairing of the controlled (loaded organic and rich electrolyte copper concentrations) and manipulated (PLS series and parallel, organic and electrolyte flow rates) variables was performed with Bristol's relative gain array analysis [Seborg et al, 2004, Skogestad and Postlethwaite, 2005]. The pairing was first studied with full non-square matrix RGA for both the transfer function matrices at frequencies  $\omega \in [0, 1/5]$ . For the loaded organic copper concentration the pairing at lower frequencies ( $\omega < 0.1$ ) favors organic flow rate, F(LO), but in higher frequencies ( $\omega > 0.1$ ) the pairing with PLS series flow rate, F(PLSS) becomes more favorable. However, the RGA values for the organic flow rate pairing do not fall below 0, so this pairing is still valid at the frequency range. For the rich electrolyte copper concentration, the RGA analysis favors pairing with electrolyte flow rate.

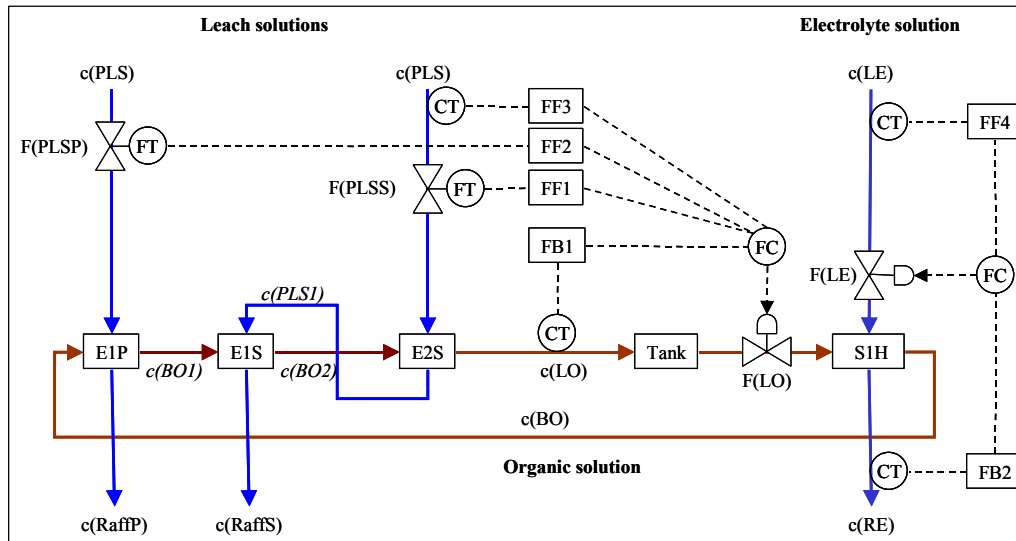
On the basis of the relative gain array analysis of the linearized dynamic model, favorable pairing of the controlled and manipulated variables for the feedback controllers is loaded organic copper concentration with organic flow rate, c(LO)-F(LO), and rich electrolyte copper concentration with electrolyte flow rate, c(RE)-F(LE). The additional manipulated variables, PLS flow rates, F(PLSS) and F(PLSP), are considered as measured disturbances. The feedforward compensators (rejecting disturbances) are constructed for the PLS series flow rate with organic flow rate, F(PLSS)-F(LO), the PLS parallel flow rate with organic flow rate, F(PLSP)-F(LO), the PLS copper concentration with organic flow rate, c(PLS)-F(LO), and lean electrolyte copper concentration with electrolyte flow rate, c(LE)-F(LE).

The single-input single-output control strategy and comparable multi-input multi-output control strategy are designed on the basis of the feedback control structure, utilizing the linearized dynamic model of the copper solvent extraction process. These strategies are alternatives for the stabilizing control level in the proposed control hierarchy.

In the single-input single-output control strategy the first control loop consists of the loaded organic copper concentration, c(LO), which is controlled by manipulating the organic flow rate, F(LO). The second control loop consists of rich electrolyte copper concentration, c(RE), which is kept in setpoint by manipulating electrolyte flow rate, F(LE). For the single-input single output strategy PI controllers are used.

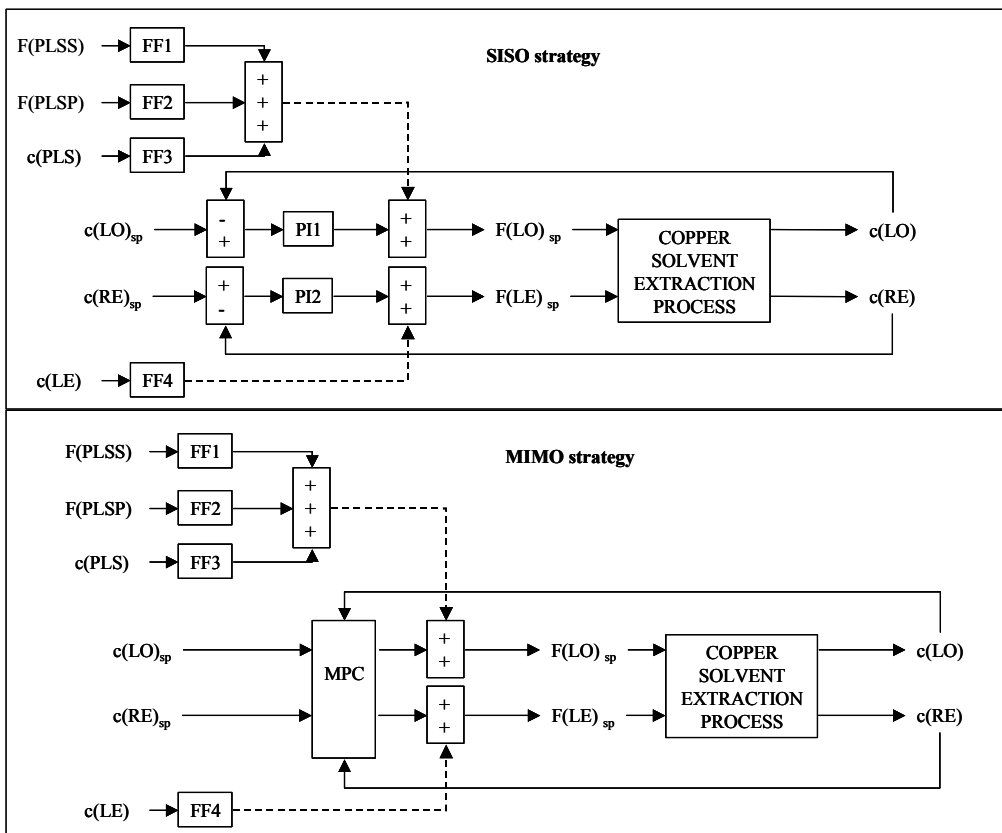
In the multi-input multi-output control strategy the model predictive controller is designed on the basis of the linearized dynamic model of the copper solvent extraction process. The two controlled variables are loaded organic copper concentration, c(LO), and rich electrolyte copper concentration, c(RE). The manipulated variables are organic flow rate, F(LO), and electrolyte flow rate, F(LE).

The feedforward controllers are constructed to compensate for changes of PLS and lean electrolyte concentrations, c(PLS) and c(LE), and PLS series and parallel flow rates, F(PLSS) and F(PLSP), as shown in Fig. 5. The feedforward compensators are of the lead-lag type.



**Fig. 5.** Control strategy: the loaded organic copper concentration,  $c(LO)$ , is feedback controlled (FB1) with organic flow  $F(LO)$ . The rich electrolyte copper concentration,  $c(RE)$ , is feedback controlled (FB2) with electrolyte flow,  $F(LE)$ . The disturbances are compensated with feedforward controllers, for PLS series flow,  $F(PLSS)$  with FF1, for PLS parallel flow,  $F(PLSP)$ , with FF2, for PLS copper concentration,  $c(PLS)$ , with FF3, and lean electrolyte copper concentration,  $c(LE)$  with FF4.

The difference between the single input-single output and multi-input multi-output control strategies is illustrated in Fig. The compensation of input concentration disturbances and set-point tracking of the loaded organic and rich electrolyte concentrations is done by changing the organic and electrolyte flow rates. Change in the flow rates changes organic to aqueous ratio in mixers and thus changes the output concentration of the process. An effective way to compensate concentration disturbances would be to change the reagent volume percent in the organic solution, but due to lack of instrumentation and measurements, this is not currently a realizable approach.



**Fig. 6.** The single-input single-output (SISO) and multi-input multi-output (MIMO) control strategies. The SISO strategy utilizes two PI controllers marked with PI1 and PI2, and the MIMO strategy utilizes model predictive controller, marked with MPC. The additional feedforward compensators are marked with FF. The setpoints are marked with subscript sp.

The two PI-controllers were tuned with the internal model control (IMC) rules. In order to take into account the effects of the loop interactions, MIMO Nyquist stability criteria is tested, as suggested by Skogestad and Postlethwaite [18]. The determinant was calculated for the transfer function of the system with the controllers,  $\det(I+G_p(s) G_c(s))$ , where the time delays were estimated with first order padé approximations. The Nyquist plot of the determinant does not encircle origin, therefore the process with PI controllers is stable.

The model predictive controller is based on the eight order state space model (for MPC see for example Maciejowski [19]). The cost function to be minimized has the following form:

$$J = \sum_{i=n1}^{n2} \gamma_{yi} [y_r(k+d+i) - \hat{y}(k+u+i | k)]^2 + \sum_{j=1}^{nu} \gamma_{uj} \Delta u^2(k-1+j) \quad (22)$$

where  $y_r$  is the reference trajectory,  $\hat{y}$  is the output prediction and  $u$  is the input value and  $k$  is the present moment and  $d$  is the discrete dead time. The tuning parameters are the following: the prediction horizon ( $n2-n1$ ), the control horizon  $nu$ , and the weights  $\gamma_y$  and  $\gamma_u$ . The output weight  $\gamma_y$  is penalizing the error between the output and the reference trajectory, and  $\gamma_u$  is penalizing the changes in the manipulated variable. The MPC tuning was done by adjusting the following parameters: prediction horizon ( $n2-n1$ ), control horizon ( $nu$ ), cost function weights for: controlled variable weights ( $\gamma_y$ ) and manipulated variable rate weights ( $\gamma_u$ ). Hard constraints were not assigned. Initial choices for the parameters were the following: prediction horizon was the longest settling time of the manipulated variable - controlled variable pairs, and the control horizon the larger than longest dead time of the manipulated variable-controlled variable pairs. The rich electrolyte copper concentration had larger weight than loaded organic copper concentration to emphasize the importance of the end product quality. The manipulated variable rate weights were tuned to avoid big changes and oscillating behaviour. The feedforward compensators were lead-lag type, designed by using the transfer function models between the controlled variable-disturbance variable and the controlled variable-manipulated variable. The time delay parts of the transfer function models were discarded. The feedforward controllers were added one by one on the top of the feedback controllers, and the coefficients were tuned. The controllers were tuned to have the best possible performance for disturbance rejection and setpoint tracking in different scenarios. The performances under different scenarios are compared in the following in Sections 5.1 and 5.2. For further details on the control strategy and controller tuning, please refer to Komulainen [20].

## 5 Control performance

### 5.1 Control performance for disturbance rejection

The disturbance rejection performances of the following controller combinations were tested and compared: open loop (OL) with no controllers, both of the two PI controllers (PI), both the two PI controllers with four feedforward compensators (PI+FF), model predictive controller (MPC), model predictive controller with four feedforward compensators (MPC+FF).

The testing was done by introducing  $\pm 5\%$  changes to the following the inputs and input combinations one at the time: PLS series flow rate  $F(\text{PLSS})$ , PLS parallel flow rate  $F(\text{PLSP})$ , PLS copper concentration  $c(\text{PLS})$ , lean electrolyte copper concentration  $c(\text{LE})$ , simultaneous with different signs for PLS series flow rate and PLS copper concentration  $F(\text{PLSS})$  and  $c(\text{PLS})$ , simultaneous with same signs for PLS parallel flow rate and PLS copper concentration  $F(\text{PLSP})$  and  $c(\text{PLS})$ , simultaneous for PLS copper concentration and electrolyte copper concentration  $c(\text{PLS})$  and  $c(\text{LE})$ . The measure to compare the controller combinations was integral of absolute error (IAE) between the constant setpoints of the controlled variables and the outputs of the controlled variables under control.

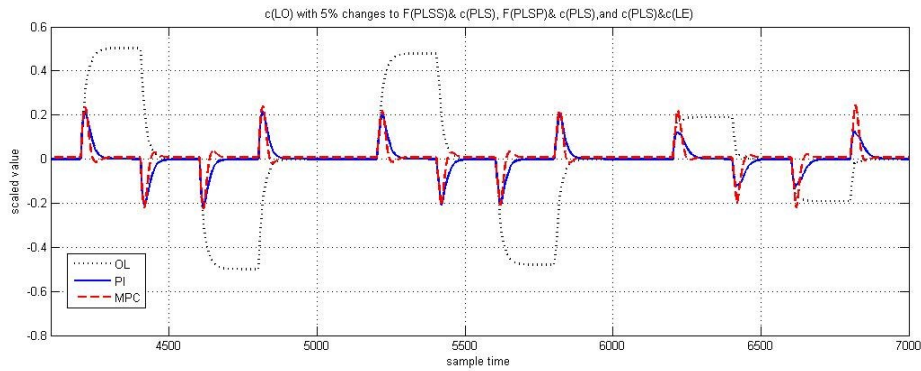
The results with the model predictive controller were better for disturbance rejection than with PI-controllers for the loaded organic copper concentration, as can be seen from Table 3. With the feedforward compensators, PI controllers performed better to reject the  $F(\text{PLSS})$ ,  $F(\text{PLSP})$  and  $c(\text{PLS})$  disturbances. All the combined disturbances are rejected more efficiently with MPC than with PI controllers as shown in Fig. 7.

The disturbance rejection for rich electrolyte copper concentration is clearly better with MPC than with PI controllers, as presented in Table 3. Addition of the feedforward controller for lean electrolyte copper concentration improves the disturbance rejection characteristics for MPC. The other feedforward compensators slightly worsen the result for rich electrolyte copper concentration, but on the other hand the result for loaded organic copper concentration is significantly improved.

**Table 3**

The integral of absolute error (IAE) for loaded organic copper concentration  $c(\text{LO})$  and rich electrolyte copper concentration  $c(\text{RE})$  with disturbances.

	<b>F(PLSS)</b>	<b>F(PLSP)</b>	<b>c(PLS)</b>	<b>c(LE)</b>	<b>F(PLSS) and c(PLS)</b>	<b>F(PLSP) and c(PLS)</b>	<b>c(PLS) and c(LE)</b>
$c(\text{LO})$ OL	66.5	57.7	134.3	57.9	200.5	191.7	76.4
$c(\text{LO})$ PI	9.1	7.8	18.4	4.1	27.6	26.2	21.0
$c(\text{LO})$ PI+FF	4.0	1.9	7.1	2.6	9.9	7.1	7.2
$c(\text{LO})$ MPC	7.2	6.1	14.3	7.1	21.4	20.4	20.3
$c(\text{LO})$ MPC+FF	4.2	2.2	8.0	2.0	11.4	9.1	8.7
$c(\text{RE})$ OL	83.9	72.8	169.7	669.1	253.3	242.1	506.7
$c(\text{RE})$ PI	8.0	6.8	16.0	40.8	24.0	22.8	31.0
$c(\text{RE})$ PI+FF	7.7	5.9	15.4	7.4	22.7	21.1	19.5
$c(\text{RE})$ MPC	2.4	1.9	3.3	28.5	5.6	4.9	29.5
$c(\text{RE})$ MPC+FF	4.9	2.9	6.7	5.5	11.2	9.5	8.9



**Fig. 7.** Loaded organic copper concentration with +5% change to  $c(\text{PLS})$  and  $F(\text{PLSS})$ ,  $c(\text{PLS})$  and  $F(\text{PLSP})$ , and  $c(\text{PLS})$  and  $c(\text{LE})$ , under open loop control (OL), PI controllers (PI), and model predictive controller (MPC).

## 5.2 Control performance for setpoint tracking

The setpoint tracking performances of the controller combinations listed in Section 5.1 were tested and compared. The testing was done by introducing  $\pm 5\%$  changes to the setpoints of the controlled variables and their combination: loaded organic copper concentration  $c(\text{LO})$ , rich electrolyte copper concentration  $c(\text{RE})$ , simultaneous with same signs for loaded organic copper concentration and rich electrolyte copper concentration  $c(\text{LO})$  and  $c(\text{RE})$ . The measure to compare the controller combinations was integral of absolute error (IAE) between the setpoints of the controlled variables and the outputs of the controlled variables under control.

The results with the model predictive controller were better for loaded organic copper concentration and rich electrolyte copper concentration setpoint tracking than with PI-controllers, as can be seen from Table 4. MPC was able to minimize the control loop interactions, and the overall performance was better than with PI-controllers. The controller interaction is significantly decreased with MPC, for example the setpoint tracking of the rich electrolyte copper concentration  $c(\text{RE})$  has value 4.8 with MPC and 15.7 with PI-controllers. Within the MPC structure the rich electrolyte copper concentration setpoint tracking had higher coefficient than loaded organic copper concentration, and thus here the loop interaction plays against the good result for the loaded organic copper concentration. MPC provides better setpoint tracking than PI-controllers. Adaptation to the new loaded organic setpoint is fast with small overshoot. The simultaneous setpoint change for the both controlled variables results in similar responses between MPC and PI controller, however, as can be seen from Table 5 and Fig. 13, MPC is performing slightly better than PI controllers (29.1 and 30.2 correspondingly). Also the effect of the rich electrolyte copper concentration setpoint change is far smaller with model predictive controller than with PI controllers (IAE 4.8 and 15.7 correspondingly).

**Table 4**

The integral of absolute error (IAE) for loaded organic copper concentration  $c(LO)$  and for rich electrolyte copper concentration  $c(RE)$  with set point tracking.

	<b>c(LO)</b>	<b>c(RE)</b>	<b>c(LO) and c(RE)</b>
c(LO) OL	191.4	0.0	191.4
c(LO) PI	30.8	15.7	30.2
c(LO) MPC	25.7	4.8	29.1
c(RE) OL	0.0	906.0	906.0
c(RE) PI	19.9	64.4	81.5
c(RE) PI+FF	19.9	64.4	81.5
c(RE) MPC	7.7	58.5	58.4
c(RE) MPC+FF	8.1	58.4	58.0

**Table 5**

The production increase, and variation decrease for rich electrolyte and loaded organic copper concentration with the control system compared to the manual control.

	<b>Production</b>	<b>AAE c(RE)</b>	<b>AAE c(LO)</b>
PI	+4.93%	-75.15%	-71.09%
PI+FF	+4.92%	-84.11%	-80.43%
MPC	+4.93%	-81.33%	-71.20%
MPC+FF	+4.93%	-91.75%	-73.49%

### 5.3 Comparison of the control strategies to the manual control strategy

In order to verify the benefits of the proposed single-input single-output and multi-input multi-output control strategies, the performances of the proposed control strategies are compared to the performance of the manual control strategy of the case copper solvent extraction plant. The control strategies are compared to each other by the average copper production and the average absolute error around the setpoints on the basis of the simulated and measured outputs.

The average production is determined as follows:

$$prod = \frac{\sum_{i=1}^N (c(RE)(i) - c(LE)(i)) \cdot F(LE)(i)}{N} \quad (13)$$

where  $N$  is the total number of samples.

The average absolute error between output copper concentration  $c$  and the setpoint  $c_{sp}$  is calculated as follows:

$$AAE = \frac{\sum_{i=1}^N |c_{sp}(i) - c(i)|}{N} \quad (24)$$

The industrial measurements are representing the manual control strategy. The setpoints for the controlled variables are determined as one day moving average of the corresponding industrial online measurement.

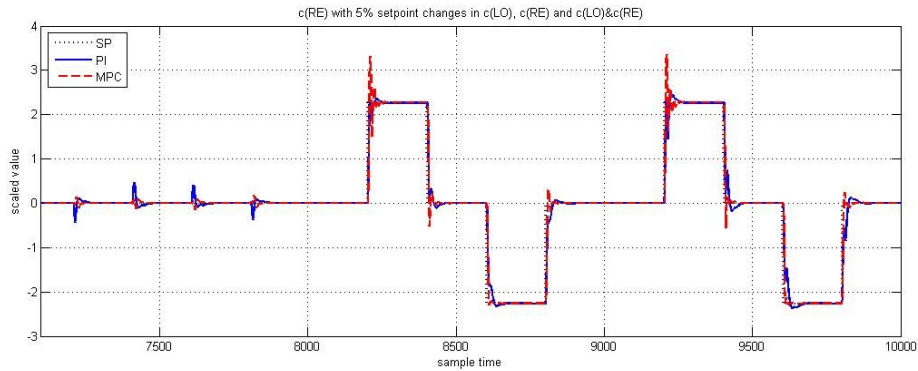
For the single-input single-output and multi-input-multi-output control strategies, the inputs to the simulator with the controllers are the industrial measurements and adapted parameters for equilibrium isotherm and efficiency correction parameters. The setpoints for the controlled variables are determined by solving the optimization problem with the offline process data.

In the optimization the minimum and maximum flow rates of the industrial data are considered as additional constraints. Also the organic to aqueous ratios are determined on the basis of the maximum value for the extraction and the minimum value for the stripping from the industrial data.

No assumptions are made of the operation of the electrowinning, but the lean electrolyte copper concentration measurements are used as such. This assumption might affect the control results negatively, since the rich electrolyte control loop has to do extra work to stabilize the disturbances from lean electrolyte copper concentration.

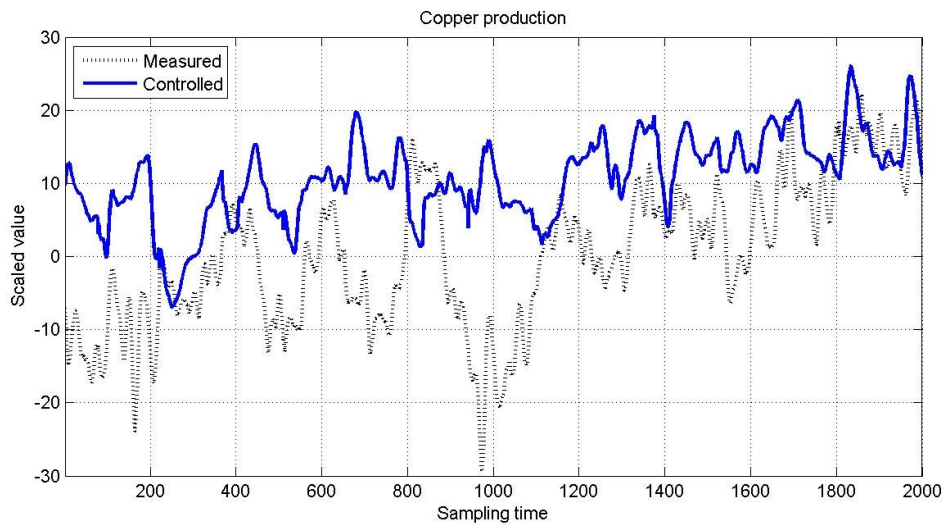
The production increase with the different control strategies compared to the manual control is presented in Table 5. Compared to the industrial test data set, the production was increased almost 5% with all the control strategies. The production increase is due to the efficient optimization and therefore the percentages with all the control strategies are similar.

For the first data set, the production under the PI controllers with the feedforward compensator is presented in Fig. 8: rich electrolyte copper concentration with +5% changes to  $c(\text{LO})$  setpoint,  $c(\text{RE})$  setpoint, and simultaneously to both setpoints, under open loop control (OL), PI controllers (PI), and model predictive controller (MPC).

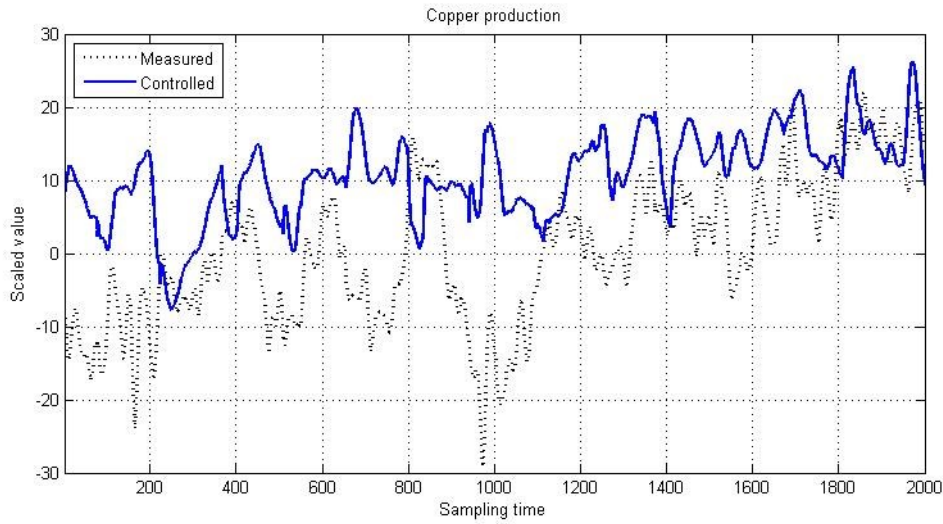


**Fig. 8.** Rich electrolyte copper concentration with +5% changes to  $c(\text{LO})$  setpoint,  $c(\text{RE})$  setpoint, and simultaneously to both setpoints, under open loop control (OL), PI controllers (PI), and model predictive controller (MPC).

Fig. 9, and under MPC control with the  $c(\text{LE})$  feedforward compensator is shown in Fig. 10. The production is clearly higher and has less variation compared to the manual operating practice. The visual comparison shows that the model predictive controller also causes less variation to the production compared to the PI controllers with the feedforward compensators.



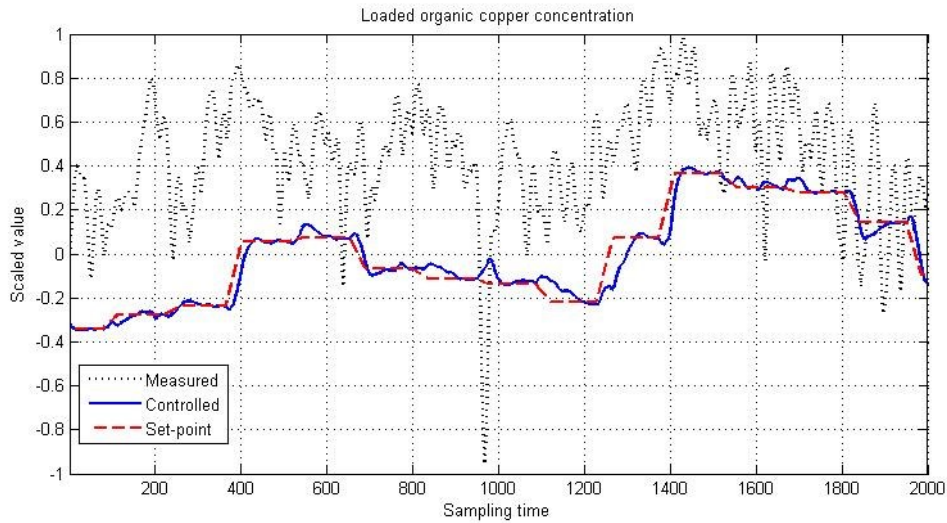
**Fig. 9.** Copper production: industrial measurement (dotted) and PI+FF controlled in simulation environment (solid).



**Fig. 1.** Copper production: industrial measurement (dotted) and MPC+FF controlled in simulation environment (solid).

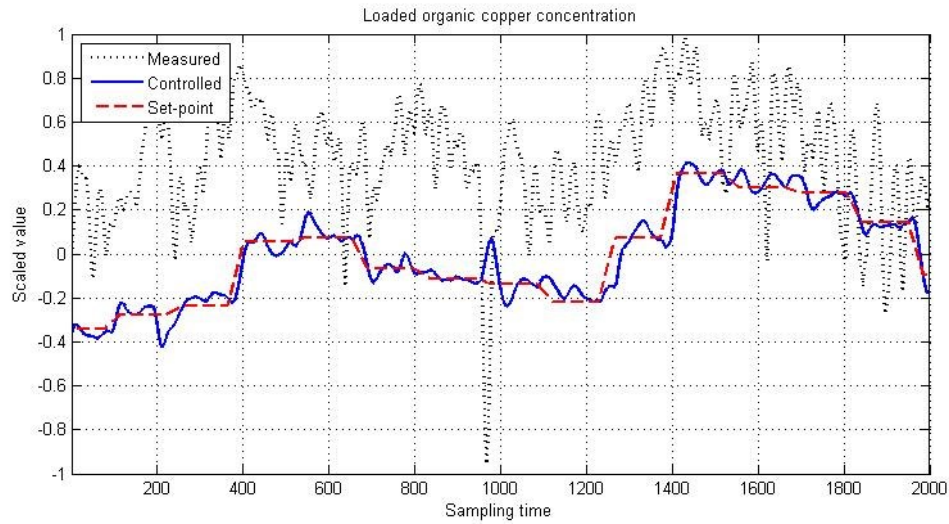
The variation decrease with the different control strategies compared to the manual control is presented in Table 5. The variation in the controlled variables is decreased between 70-90%. For the rich electrolyte copper concentration the decrease is up to 90% and for the loaded organic copper concentration up to 80%. The difference between the controllers is mainly due to the tuning of the controllers. The MPC is tuned to track the rich electrolyte copper concentration better than loaded organic copper concentration.

The PI controller with the feedforward compensators tracks better the setpoint for the loaded organic copper concentration, as shown in Fig. 11, than the MPC with the feedforward compensator, as presented in Fig. 12. The optimization for this data set reduces the setpoint of the loaded organic copper concentration to lower level than the measurement, but the flow rate is higher and thus, more copper is transferred from pregnant leach solution to the organic solution.



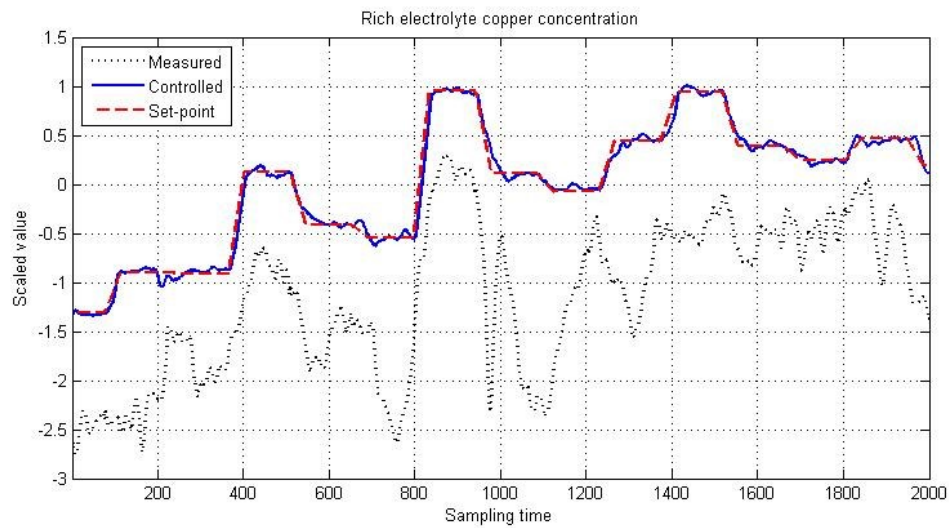
**Fig. 11.** Loaded organic copper concentration: industrial measurement (dotted, black), PI+FF controlled in simulation environment (solid, blue), and setpoint (red, dashed).





**Fig. 12.** Loaded organic copper concentration: industrial measurement (dotted, black), MPC+FF controlled in simulation environment (solid, blue), and setpoint (red, dashed).

The setpoint tracking for the rich electrolyte copper concentration is more efficient with the MPC with feedforward compensator, as shown in Fig. 13, than with the PI controllers with feedforward compensators, as presented in Fig. 14. The rich electrolyte copper concentration setpoint is higher than the measurement, but the flow rate slightly lower.



**Fig. 13.** Rich electrolyte copper concentration: industrial measurement (dotted, black), PI+FF controlled in simulation environment (solid, blue), and setpoint (red, dashed).

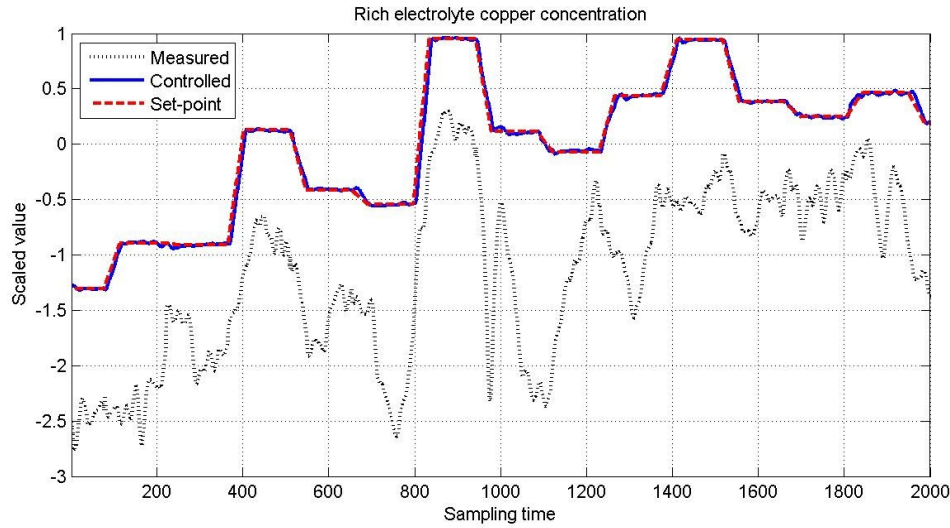


Fig. 14. Rich electrolyte copper concentration: industrial measurement (dotted, black), MPC+FF controlled in simulation environment (solid, blue), and setpoint (red, dashed).

## 6 Conclusions

A two level control strategy that stabilizes and optimizes the production of an industrial copper solvent extraction process was designed and tested. First, the linear transfer function and state space models were obtained to follow adequately well the trends of the industrial data sets. The controlled variable – manipulated variable pairing was performed using the relative gain array (RGA). The optimization algorithm was developed on the basis of the linear process models. Next, the SISO and MIMO controllers were designed on the basis of the linear process models. Both control strategies were tested for setpoint tracking and disturbance rejection with successful results.

The controllers' performances were compared to the manual control practice in simulation environment with the two industrial data sets. The benefits of the control system were verified by comparing the variation of the controlled variables and the produced copper tons. With PI controllers the variation in the rich electrolyte copper concentration was decreased by 75-85% and with MPC the decrease was around 80 - 90% on average. The copper mass production was increased with about 5% with both controllers. The control results are very encouraging for the further testing of the control system on the industrial copper solvent extraction plant.

## Acknowledgements

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## List of symbols and abbreviations

### LIST OF THE SYMBOLS

$a_{ij}, b_{ij}, c_{ij}$	parameters of the transfer function model	
$A, B$	parameters of extraction equilibrium isotherm	
$C, D$	parameters of stripping equilibrium isotherm	
$c$	concentration of copper if not otherwise specified	(g/l)
$c^*$	equilibrium concentration	(g/l)
$c^{aq}$	copper concentration of aqueous solution	(g/l)
$c^{org}$	copper concentration of organic solution	(g/l)
$f$	general function	
$F$	flow rate	(m <sup>3</sup> /min)
$G_{p, tot}$	total transfer function of the process	
$K_i$	mass transfer coefficient, i=E1,E2,EP,S	(1/min)
$M_i$	matrix	
$N$	number of samples	
$P$	profit function of the optimization problem	
$r_i$	steady state gain of a transfer function	
$s$	Laplace transform variable	
$t$	time	(min)

$V$	volume	(m <sup>3</sup> )
$V_m$	volume of the mixer	(m <sup>3</sup> )
$V_s$	volume of the settler	(m <sup>3</sup> )
$w_i$	steady state gain of a transfer function	
$Y_{meas}$	measured value	
$Y_{model}$	model prediction	
$\alpha_i$	Extraction / stripping efficiency, i=E1,E2,EP,S	
$\beta_1, \beta_2$	the organic to aqueous flow ratio constraints	
$\kappa_i$	Equilibrium constant, i=E1,E2,EP,S	
$\omega$	frequency	(1/rad)
aq	aqueous	
el	electrolyte	
max	maximum value	
min	minimum value	
opt	optimum value	
org	organic	
out	out of mixer / unit	
outs	out of settler	

#### ABBREVIATIONS

AAE	average absolute error
acid	acidity of the electrolyte solution
BO	Barren organic solution
c(BO)	copper concentration of the barren organic solution
c(LE)	copper concentration of the lean electrolyte solution
c(LO)	copper concentration of the loaded organic solution
c(PLS)	copper concentration of the pregnant leach solution
c(Raff)	copper concentration of the raffinate solution
c(RE)	copper concentration of the rich electrolyte solution
Cu	copper
CV	controlled variable
DV	disturbance variable
E	extraction unit
E1S,E2S,E1P	Series 1 and 2 and parallel extraction units
EW	electrowinning
F(LE)	flow rate of the electrolyte solution
F(LO)	flow rate of the organic solution
F(PLS)	flow rate of the pregnant leach solution
FB	feedback
FF	feedforward
FOPTD	first order plus time delay model
H <sup>+</sup>	hydrogen ion
IAE	integral of the absolute error index
[IN]	vector of the input variables
ISE	integral of the square error index
LE	lean electrolyte solution
LO	loaded organic solution
MIMO	multi input – multi output
MPC	model predictive control
MV	manipulated variable
OL	open loop
[OUT]	vector of the output variables
pH	pH level of the pregnant leach solution
PI	PI (proportional, integral) controller
PLS	pregnant leach solution
Raff	raffinate solution of the extraction step
RaffS	raffinate solution from series extraction step
RaffP	raffinate solution from parallel extraction step
RE	rich electrolyte solution
RGA	relative gain array method
S/S1H	stripping unit

SISO	single input – single output
SS	state space model
SX	solvent extraction
TF	transfer function model
vol	reagent volume percent in the organic solution

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